

SPONTANEOUS LORENTZ VIOLATION, NAMBU-GOLDSTONE MODES, AND MASSIVE MODES

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In any theory with spontaneous symmetry breaking, it is important to account for the massless Nambu-Goldstone and massive Higgs modes. In this short review, the fate of these modes is examined for the case of a bumblebee model, in which Lorentz symmetry is spontaneously broken.

1 Spontaneous Lorentz violation

Spontaneous symmetry breaking has three well-known consequences. The first is Goldstone's theorem, which states that massless Nambu-Goldstone (NG) modes should appear when a continuous symmetry is spontaneously broken. The second is that massive Higgs modes can appear, consisting of excitations out of the degenerate minimum. The third is the Higgs mechanism, which can occur when the broken symmetry is local.

In conventional particle physics, these processes involve a scalar field with a nonzero vacuum expectation value that induces spontaneous breaking of gauge symmetry. However, in this paper, these processes are examined for the case where it is a vector field that has a nonzero vacuum value and where the symmetry that is spontaneously broken is Lorentz symmetry.

The idea of spontaneous Lorentz violation is important in quantum-gravity theory. For example, in string field theory, mechanisms have been found that can lead to spontaneous Lorentz violation.¹ It has also been shown that spontaneous Lorentz breaking is compatible with geometrical consistency conditions in gravity, while explicit Lorentz breaking is not.²

The Standard-Model Extension (SME) describes Lorentz-violating interactions at the level of effective field theory.^{3,4} In recent years, numerous tests of Lorentz symmetry have been performed, and sensitivities to extremely small effects has been attained.⁵

2 Bumblebee models

The simplest example of a theory with spontaneous Lorentz violation is a bumblebee model.^{1,2,6,7,8,9,10,11,12,13,14,15,16} It is defined as an effective field theory with a vector field B_μ that has a nonzero vacuum value, $\langle B_\mu \rangle = b_\mu$. In general, the theory includes gravity with a metric field $g_{\mu\nu}$. The interactions in the theory can be written in terms of a potential V , which has a minimum when B_μ and $g_{\mu\nu}$ equal their vacuum values.

Bumblebee models are invariant under both diffeomorphisms and local Lorentz transformations. To reveal the local Lorentz symmetry, a vierbein formalism is used. The vierbein e_μ^a relates tensor fields defined on the spacetime manifold, e.g., $g_{\mu\nu}$ and B_μ , to the corresponding fields defined in a local Lorentz frame, e.g., η_{ab} and B_a .

Spontaneous Lorentz violation occurs when the vector field has a nonzero vacuum solution, b_a , in a local Lorentz frame. Since the metric and vierbein have background values in the spacetime frame, the bumblebee vector in the spacetime frame also has a nonzero vacuum value, $b_\mu = \langle e_\mu^a \rangle b_a$, which therefore spontaneously breaks diffeomorphism symmetry. This illustrates a general result that whenever local Lorentz symmetry is spontaneously broken by a local background tensor, then diffeomorphism symmetry is spontaneously broken as well¹⁰.

A variety of bumblebee models can be defined depending on the definitions of the potential V and kinetic terms for $g_{\mu\nu}$ and B_μ . If an Einstein-Hilbert action is chosen for the gravitational sector, then the excitations of the metric include the usual graviton modes as in general relativity. The NG modes can be identified as virtual symmetry transformations about the vacuum solution that stay in the minimum of the potential V , while the massive modes are excitations that move out of the minimum.

The interpretation of the NG modes depends on the choice of kinetic terms for the bumblebee field. In one class of models, the bumblebee field is treated as a vector in a vector-tensor theory of gravity, and a Will-Nordtvedt form of kinetic term is used.¹⁷ In this scenario, the NG modes are considered additional gravitational excitations. An alternative interpretation is to treat the field B_μ as a generalized vector potential. In this case, a Maxwell kinetic term is used, and the NG modes are interpreted as photons.^{6,7,10,11}

The original model of Kostelecký and Samuel (KS) uses a Maxwell kinetic term for B_μ and a potential V that has a minimum when the condition $B_\mu B^\mu = \pm b^2$ holds.¹ The KS bumblebee model in a Minkowski background

has Lagrangian

$$\mathcal{L}_{\text{KS}} = \frac{1}{16\pi G}R - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} - V + B_\mu J^\mu + \mathcal{L}_M. \quad (1)$$

Here, $B_{\mu\nu} = D_\mu B_\nu - D_\nu B_\mu$, is the generalized field strength defined using gravitational covariant derivatives, J^μ is a matter current, and \mathcal{L}_M is the Lagrangian for the matter fields. A generic form for the potential V is a power-series expansion in $B_\mu B^\mu$, which when truncated to second order is

$$V = \frac{1}{2}\kappa(B_\mu B^\mu \pm b^2)^2, \quad (2)$$

where κ is a constant.

A important feature of all bumblebee models, including the KS model, is that they do not have local $U(1)$ gauge symmetry, which is broken explicitly by the potential V . However, with conventional matter couplings a global $U(1)$ symmetry can exist. This results in conservation of charge in the matter sector, as given by the equation, $D_\mu J^\mu = 0$. This conservation law also ensures that the bumblebee model is perturbatively stable about the minimum solution.¹²

3 Nambu-Goldstone and massive modes

The field equations are obtained by varying \mathcal{L}_{KS} with respect to $g_{\mu\nu}$ and B_μ . These can be used to determine the propagating gravitational, NG, and massive modes.^{10,11} The excitations of the metric about the vacuum can be written as $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$, which includes the usual graviton modes. A convenient gauge choice for decoupling the NG modes is the condition $b^\mu h_{\mu\nu} = 0$. Imposing it allows the bumblebee field to be written as

$$B_\mu = b_\mu + A_\mu + \hat{b}_\mu \beta, \quad (3)$$

where \hat{b} is a unit vector along the direction of b_μ . In this expansion, A_μ is transverse to b_μ , obeying the condition $b^\mu A_\mu = 0$, while β is along the direction of b_μ .

The excitations A_μ are the NG modes generated by the broken Lorentz transformations. For these excitations, V remains in the minimum of the potential. The condition $b^\mu A_\mu = 0$ has the form of an axial gauge-fixing constraint. Of the three remaining components in A_μ , two propagate like photons in a fixed axial gauge, while the third is an auxiliary field due to a constraint in the form of a modified version of Gauss' law.

The excitation β along the direction of \hat{b}_μ is a massive-mode (or Higgs excitation). It can be written as

$$\beta = \frac{\hat{b}^\mu (B_\mu - b_\mu)}{\hat{b}^\alpha \hat{b}_\alpha}. \quad (4)$$

These excitations do not stay in the potential minimum. At leading order, the massive mode β does not propagate as a free field. Instead, it acts as a background source of both charge and energy density.^{10,11} The massive mode in the KS bumblebee model is an extra degree of freedom compared to Einstein-Maxwell theory.¹² Its behavior depends on the initial conditions that are specified for it. In particular, it has been shown that if the massive mode vanishes initially, it vanishes for all time, and the KS bumblebee model reduces to Einstein-Maxwell theory with photons as NG modes. If instead, excitations of the massive mode β are allowed, then an alternative theory to Einstein-Maxwell theory results, which could be of interest in attempts to find explanations of dark matter or dark energy.

For example, it is found that both the electromagnetic and gravitational potentials for a point particle with mass m and charge q are modified by the presence of the massive mode. The specific forms of the modified potentials depend on the initial value of the massive mode, and there are therefore numerous cases that could be explored. However, in the large-mass limit (e.g., approaching the Planck scale), excitation of the massive mode is highly suppressed, and the static potentials approach the conventional Coulomb and Newtonian forms. As a result, it is found that the usual Einstein-Maxwell solutions (describing both propagating photons and the usual static potentials) can emerge from the KS bumblebee model, despite the absence of local U(1) gauge invariance.

4 Gravitational Higgs mechanisms

Since in the context of gravity, Lorentz symmetry is a local symmetry, the possibility of a Higgs mechanism occurs as well. However, with two sets of broken symmetries, local Lorentz transformations and diffeomorphisms, there are actually two types of Higgs mechanisms to consider.

In the original paper by Kostelecký and Samuel, it was shown that a conventional Higgs mechanism involving the metric does not occur.¹ This is because the quadratic term that is generated by covariant derivatives involves the connection, which consists of derivatives of the metric and not the metric itself. As a result, no mass term for the metric is generated according to the usual Higgs mechanism.

However, more recently, it has been shown that a Higgs mechanism can also occur that involves the spin connection,^{10,11} which appears in covariant derivatives acting on local tensor components. When a local tensor has a vacuum value, quadratic mass terms for the spin connection can be generated. However, a viable Higgs mechanism can occur only if the spin connection is a dynamical field. This requires that there is torsion in the theory and that the

geometry is Riemann-Cartan. As a result, a conventional Higgs mechanism for the spin connection is possible, but only in a Riemann-Cartan geometry. If torsion is permitted, a number of new types of models with Lorentz breaking can be explored. However, finding a viable model free of ghosts remains a challenging and open issue.

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